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The Multilateration System

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INTRODUCTION

- The multilateration system description
  - The properties of the multilateration system
  - The time delay estimation
  - The position estimation
  - The error analysis

THE MULTILATERATION SYSTEM DESCRIPTION

- Multilateration system gets the aircraft position through three or more distributed receivers. A multilateration system is called Time Difference of Arrival system (TDOA) as well. These receivers are connected with a central unit via the communication link.

- The task of the estimation of the aircraft position can be split into the two independent parts. The first one is the time delay estimation and the second one is the position estimation. Both part are discused in the following sections.

- A modeling TDOA system for a short base passive radar system based on Automatic Dependent Surveillance-broadcast messages. Thus, a well-known parameters is estimated the maximum position error.

THE MULTILATERATION SYSTEM DESCRIPTION

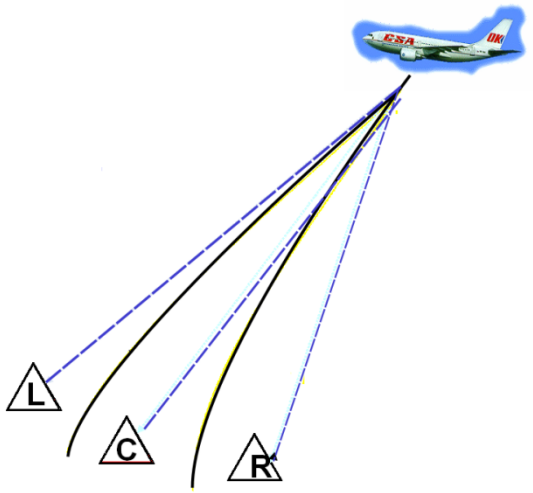
For known receivers positions [x_j, y_j, z_j] and transfer delays τ_j including the individual signal delays in the particular paths) for j =(1, 2, 3, 4) we get a set of nonlinear measurement equations for the unknown aircraft position [x_0, y_0, z_0].

$$t_j - t_i = \frac{R_j - R_i}{c} + \left(\tau_j - \tau_i\right) + \delta t_{ji}$$

Where:

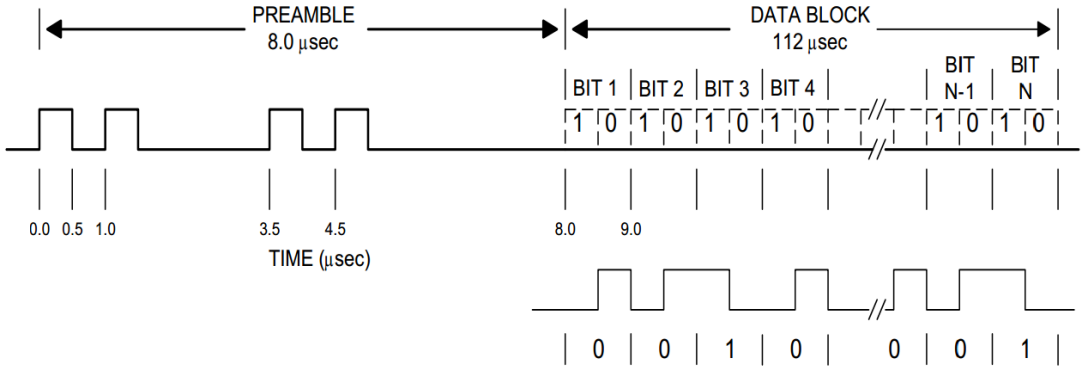
t_j	is the time of arrival, of the signal to the j-th receiver,
R_j	is a transmitter to the j-th receiver distance,
c	is the velocity of light,
δt_{ji}	is a summary TDOA measurement error.

Principle of MLAT



THE TIME DELAY ESTIMATION

- For time delay estimation is necessary to get the most precision estimator. This goal is solved by the best unbiased estimator of the time difference of arrivals of two unknown signals. It is based on a Cross-Correlation Function (CCF) of those received signals. Unfortunately the signals contains the additive noise.
- Noise has the Gaussian distribution and the noise is uncorrelated with the received signals.
- Curve fitting provides better estimation of CCF.
- The receive signal is shown at the figure.



Example: Message Data Block Waveform Corresponding to Bit Sequence 0010...001

THE POSITION ESTIMATION

- The resulting set of linear equations leads to the following estimation of the variance matrix **S** of the target position deviation vector $\delta\mathbf{r}_0$ using the Least Squares Method (LSM)

$$\mathbf{S} = (\mathbf{D}^H\mathbf{D})^{-1}\mathbf{D}^H.\mathbf{W}.\mathbf{D}.(\mathbf{D}^H\mathbf{D})^{-1}$$

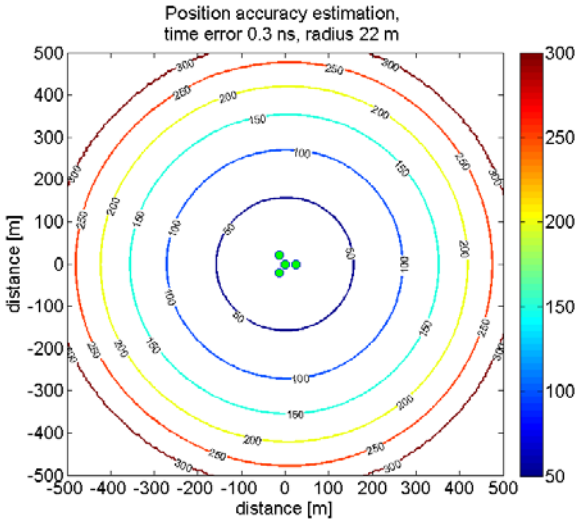
Where **D** is a differential measurement matrix and **W** is a variance matrix of measurement errors as follows:

$$\mathbf{S} = \mathrm{var}(\delta\mathbf{r}_0) \qquad \delta\mathbf{r}_0 = [\delta x_0, \delta y_0, \delta z_0,]$$
$$\mathbf{W} = \mathrm{var}(\boldsymbol{\varepsilon}); \qquad \boldsymbol{\varepsilon} = [\delta t_1, \delta t_2, \delta t_3, \delta t_4];$$

Where $\delta\mathbf{r}_0$ is a vector of target coordinates deviations, $\boldsymbol{\varepsilon}$ is a vector consisting of measurement errors δt_j of the times of arrivals t_j or their disturbances with correlations, described by the variance matrix **W**

THE POSITION ERROR ANALYSIS

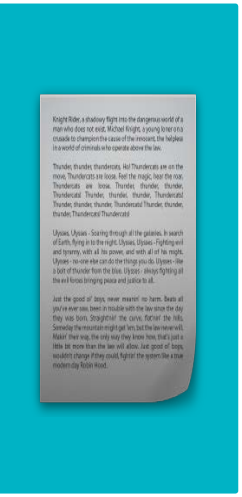
- The position error is highly influenced by the time error estimation and the receivers‘ array geometry. The shape of the array influences the error distribution for a particular direction.
- The receivers’ positions errors are not correlated with other error.
- The figure shows the error analysis for the star array geometry.
- The position error is computed using these equations.



$$\varepsilon_r = diag(\mathbf{D}.\delta\mathbf{R})$$

$$\delta\mathbf{R} = \left[\delta\mathbf{r}_1\ \delta\mathbf{r}_2\ \delta\mathbf{r}_3\ \delta\mathbf{r}_4\right]; \quad \delta\mathbf{r}_j = \left[\delta x_j, \delta y_j, \delta z_j\right]$$

ACKNOWLEDGEMENT & CONTACT



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